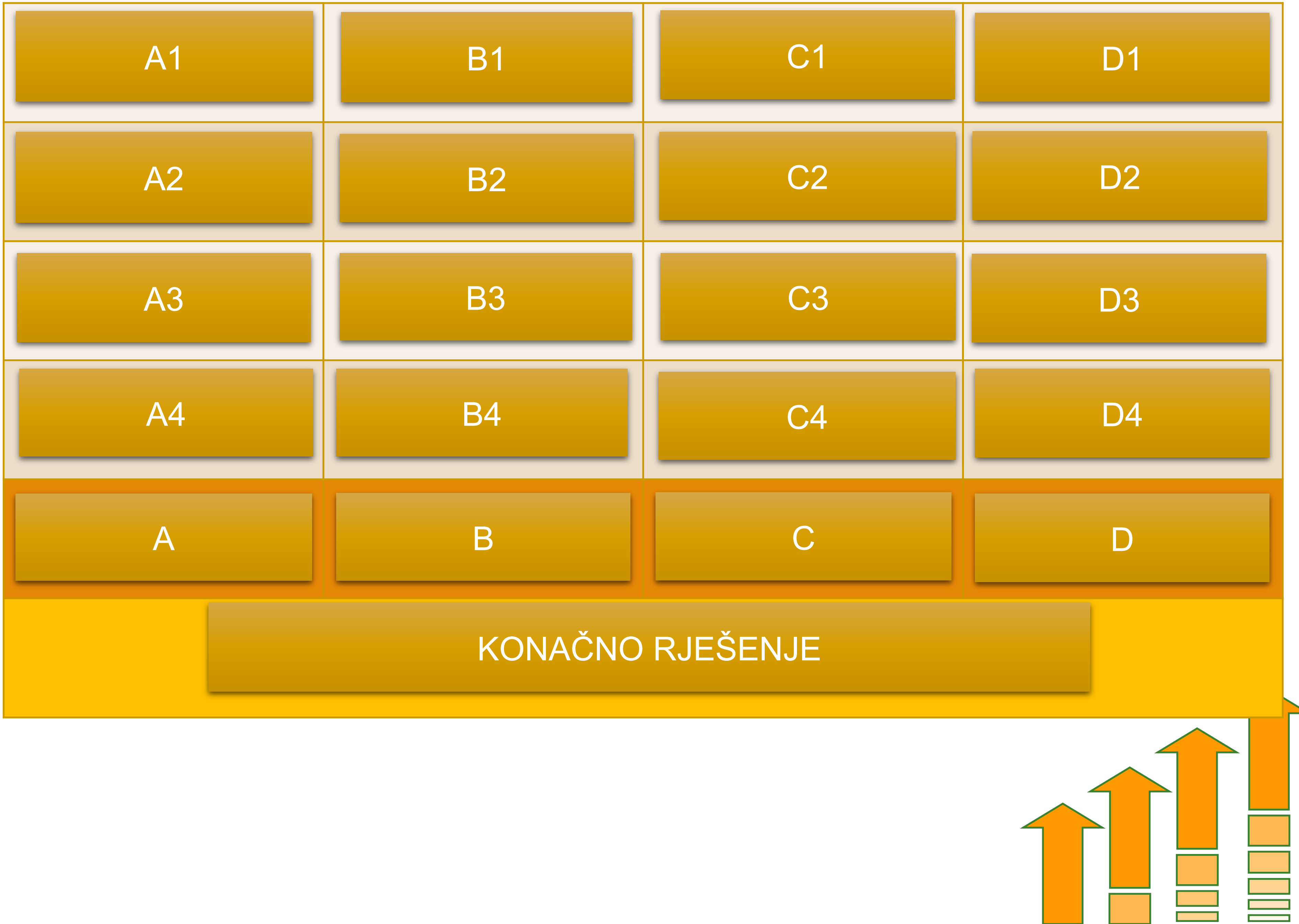
Svemogući vektori

Snježana Lukač,
prof. izvrsni savjetnik

Rebeka Kalazić,
prof. izvrsni savjetnik

Sadržaj:

uvod 1


A1
B1
C1
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A4
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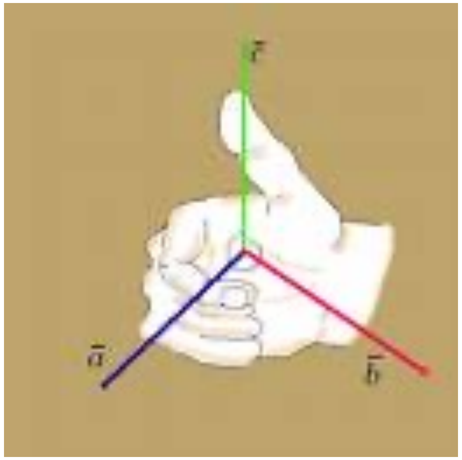
A
B
C
D
KONAČNO RJEŠENJE

Vektorski umnožak:

Za zadane nekolinearne vektore $\vec{a}$ i $\vec{b}$, vektorski umnožak $\vec{a} \times \vec{b}$ je vektor sa svojstvima:

- $|\vec{a} \times \vec{b}| = |\vec{a}|\,|\vec{b}| \sin\angle(\vec{a},\vec{b})$
- $\vec{a} \times \vec{b}$ je okomit na vektore $\vec{a}$ i $\vec{b}$
- Trojka $(\vec{a}, \vec{b}, \vec{a} \times \vec{b})$ čini desni sustav

Ako su vektori kolinearni njihov vektorski umnožak je nulvektor.



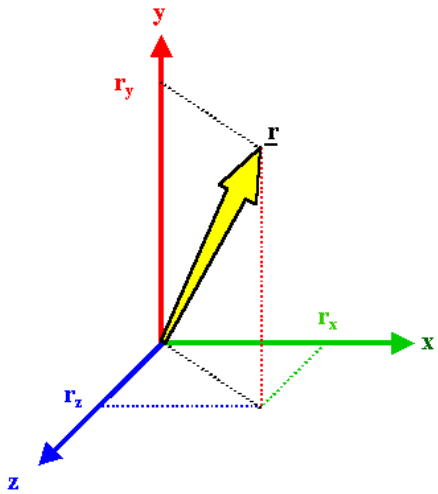
$$\vec{a} \times \vec{b} = \vec{c}$$

Svojstva vektorskog umnoška:

Za sve vektore $\vec{a}, \vec{b}, \vec{c}$ i za svaki skalar $k \in \mathbb{R}$ vrijedi:

- antikomutativnost: $\vec{b} \times \vec{a} = -\vec{a} \times \vec{b}$
- homogenost: $(k\vec{a}) \times \vec{b} = k(\vec{a} \times \vec{b})$
- distributivnost: $(\vec{a} + \vec{b}) \times \vec{c} = \vec{a} \times \vec{c} + \vec{b} \times \vec{c}$

Vektorski umnožak u koordinatnom sustavu



Tablica množenja vektora baze:

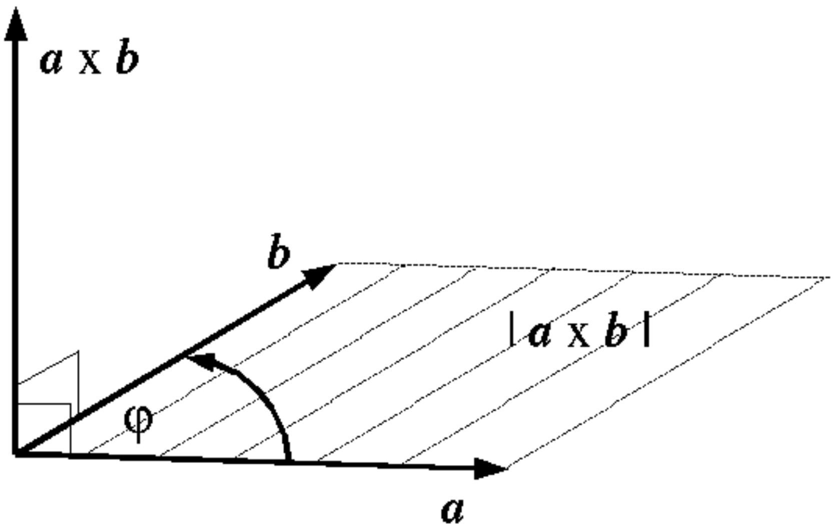
$\times$	$\vec{i}$	$\vec{j}$	$\vec{k}$
$\vec{i}$	$\vec{0}$	$\vec{k}$	$-\vec{j}$
$\vec{j}$	$-\vec{k}$	$\vec{0}$	$\vec{i}$
$\vec{k}$	$\vec{j}$	$-\vec{i}$	$\vec{0}$

Ako su vektori $\vec{a}=a_x\vec{i}+a_y\vec{j}+a_z\vec{k}$ i $\vec{b}=b_x\vec{i}+b_y\vec{j}+b_z\vec{k}$ vektorski umnožak možemo izračunati pomoću determinante:

$$\vec{a}\times\vec{b}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

Geometrijska interpretacija:

Apsolutna vrijednost vektorskog umnoška jednaka je površini paralelograma što ga zatvaraju ta dva vektora.



Skalarni umnožak:

- Skalarni umnožak vektora $\vec{a}$ i $\vec{b}$ je realan broj definiran $\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\angle(\vec{a},\vec{b})$
- U koordinatnom sustavu $\vec{a}=a_x\vec{i}+a_y\vec{j}$, $\vec{b}=b_x\vec{i}+b_y\vec{j}$ skalarni umnožak je:

$$\vec{a}\cdot\vec{b}=a_xb_x+a_yb_y$$

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Poznati poučci

Ako su $A(x_1,y_1)$, $B(x_2,y_2)$ i $C(x_3,y_3)$, tada je površina trokuta ABC jednaka

$$P=\frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$$

DOKAZ:

$$\overrightarrow{AB}=(x_2-x_1)\vec{i}+(y_2-y_1)\vec{j},\ \overrightarrow{AC}=(x_3-x_1)\vec{i}+(y_3-y_1)\vec{j}$$

$$P=\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|=\frac{1}{2}\left\|\begin{matrix}\vec{i} & \vec{j} & \vec{k}\\ x_2-x_1 & y_2-y_1 & 0\\ x_3-x_1 & y_3-y_1 & 0\end{matrix}\right\|=\frac{1}{2}\left\|\begin{matrix}x_2-x_1 & y_2-y_1\\ x_3-x_1 & y_3-y_1\end{matrix}\right\|\cdot\left|\vec{k}\right|=$$

$$=\frac{1}{2}\left|(x_2-x_1)(y_3-y_1)-(x_3-x_1)(y_2-y_1)\right|=\frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$$

Ako pravac prolazi zadanim točkama (x_1,y_1) i (x_2,y_2), tada je njegova jednadžba:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

DOKAZ: Ako su $T_1(x_1,y_1)$ i $T_2(x_2,y_2)$ zadane, a $T(x,y)$ bilo koja točka ravnine, tada uvjet da točka T leži na pravcu T_1T_2 glasi

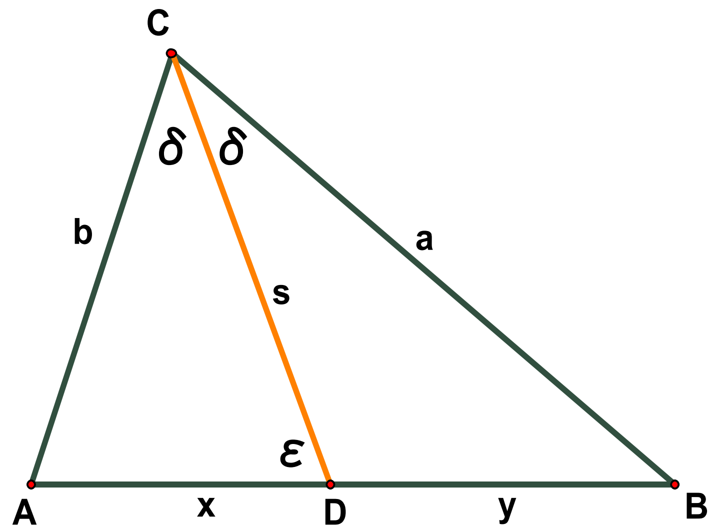
$$|\overrightarrow{T_1T_2} \times \overrightarrow{T_1T}| = 0.$$

$$\left\| \begin{matrix} \vec{i} & \vec{j} & \vec{k} \\ x_2 - x_1 & y_2 - y_1 & 0 \\ x - x_1 & y - y_1 & 0 \end{matrix} \right\| = 0 \quad \Rightarrow \quad \begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x - x_1 & y - y_1 \end{vmatrix} \cdot |\vec{k}| = 0$$

$$(x_2 - x_1)(y - y_1) - (y_2 - y_1)(x - x_1) = 0 \quad \Rightarrow \quad y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Poučak o simetrali unutarnjeg kuta trokuta:

Simetrala unutarnjeg kuta trokuta dijeli nasuprotnu stranicu u omjeru duljina drugih dviju stranica.



DOKAZ:

CD je simetrala kuta < BCA trokuta ABC,

<BCD= <DCA= δ , <ADC=ε , <CDB=180°-ε

$$P_{ADC}=\frac{1}{2}\left|\overrightarrow{CA}\times\overrightarrow{CD}\right|=\frac{1}{2}\left|\overrightarrow{DA}\times\overrightarrow{DC}\right|\to b\cdot s\cdot\sin\delta=x\cdot s\cdot\sin\varepsilon\to\frac{\sin\delta}{\sin\varepsilon}=\frac{x}{b}$$

$$P_{BCD}=\frac{1}{2}\left|\overrightarrow{CB}\times\overrightarrow{CD}\right|=\frac{1}{2}\left|\overrightarrow{DB}\times\overrightarrow{DC}\right|\to a\cdot s\cdot\sin\delta=y\cdot s\cdot\sin\varepsilon\to\frac{\sin\delta}{\sin\varepsilon}=\frac{y}{a}$$

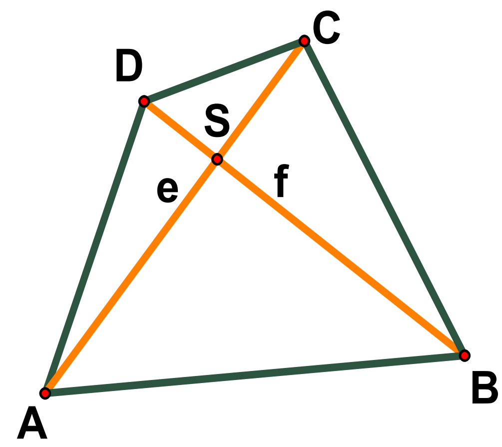
$$\longrightarrow\qquad\frac{a}{b}=\frac{y}{x}$$

Poučak o površini četverokuta:

Površina četverokuta jednaka je **$P=\frac{1}{2}ef\sin\varphi$**

gdje su **e** i **f** dijagonale četverokuta, a φ kut među njima.

DOKAZ:



$$\vec{m}=\overrightarrow{SA},\vec{n}=\overrightarrow{SC},\vec{k}=\overrightarrow{SB},\vec{l}=\overrightarrow{SD}$$

$$\angle ASD=\angle BSC=\varphi$$

$$\angle ASB=\angle CSD=180^\circ-\varphi$$

$$P=\frac{1}{2}\left|\vec{m}\times\vec{k}\right|+\frac{1}{2}\left|\vec{k}\times\vec{n}\right|+\frac{1}{2}\left|\vec{n}\times\vec{l}\right|+\frac{1}{2}\left|\vec{l}\times\vec{m}\right|$$

$$P=\frac{1}{2}mk\sin\left(180^\circ-\varphi\right)+\frac{1}{2}kn\sin\varphi+\frac{1}{2}nl\sin\left(180^\circ-\varphi\right)+\frac{1}{2}lm\sin\varphi$$

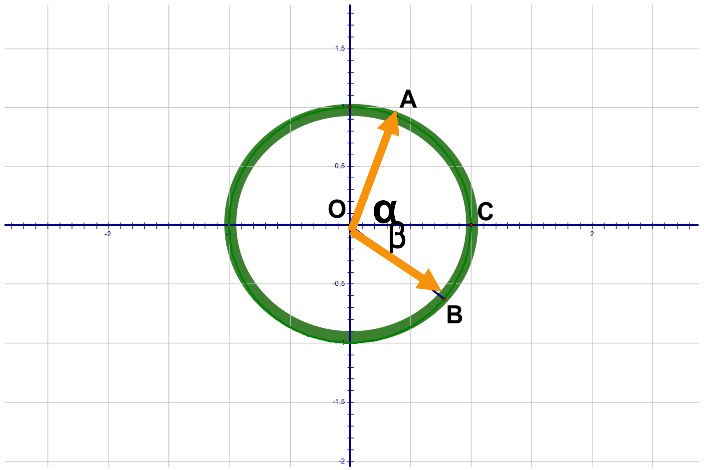
$$P=\frac{1}{2}\left(mk+kn+nl+lm\right)\sin\varphi=\frac{1}{2}\left(m+n\right)\left(k+l\right)\sin\varphi$$

$$P=\frac{1}{2}ef\sin\varphi$$

Adicijska formula za sinus:

Za bilo koja dva kuta α i β vrijedi

sin(α+β)=sinαcosβ+cosαsinβ



DOKAZ:

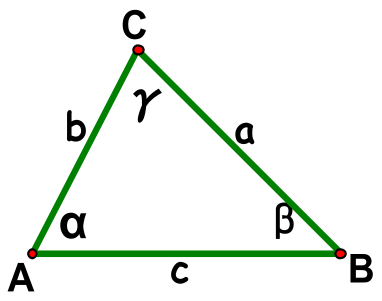
<AOC=α , <BOC=β

$$\sin(\alpha+\beta)=\frac{\left|\overrightarrow{OB}\times\overrightarrow{OA}\right|}{\left|\overrightarrow{OB}\right|\cdot\left|\overrightarrow{OA}\right|}$$

$$\overrightarrow{OB}\times\overrightarrow{OA}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \cos\beta & -\sin\beta & 0\\ \cos\alpha & \sin\alpha & 0\end{vmatrix}=\begin{vmatrix}\cos\beta & -\sin\beta\\ \cos\alpha & \sin\alpha\end{vmatrix}\cdot\vec{k}==(\sin\alpha\cos\beta+\cos\alpha\sin\beta)\cdot\vec{k}$$

$$\left|\vec{k}\right|=1,\left|\overrightarrow{OB}\right|=1,\left|\overrightarrow{OC}\right|=1\rightarrow\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$

Poučak o sinusima: Duljine stranica trokuta odnose se kao sinusi nasuprotnih kutova trokuta.



DOKAZ:

$$\vec{a}=\vec{b}+\vec{c} \rightarrow \vec{b}\times\vec{a}=\vec{b}\times(\vec{b}+\vec{c})=\vec{b}\times\vec{b}+\vec{b}\times\vec{c}=\vec{b}\times\vec{c}$$

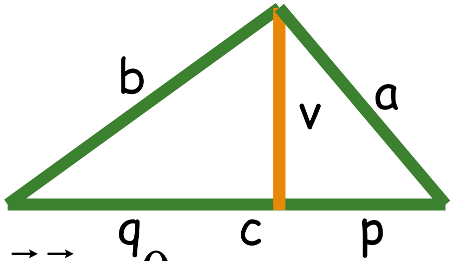
$$\rightarrow \left|\vec{b}\times\vec{a}\right|=\left|\vec{b}\times\vec{c}\right| \rightarrow \left|\vec{b}\right|\cdot\left|\vec{a}\right|\cdot\sin\angle(\vec{b},\vec{a})=\left|\vec{b}\right|\cdot\left|\vec{c}\right|\cdot\sin\angle(\vec{b},\vec{c})$$

$$\angle(\vec{b},\vec{a})=\gamma, \angle(\vec{b},\vec{c})=180^\circ-\alpha, \sin(180^\circ-\alpha)=\sin\alpha$$

$$a\cdot\sin\gamma=c\cdot\sin\alpha \rightarrow \underline{a:c=\sin\alpha:\sin\gamma}$$

Euklidov poučak: U svakom pravokutnom trokutu vrijedi: $v=\sqrt{pq}, a=\sqrt{cp}, b=\sqrt{cq}$
gdje je v visina na hipotenuzu c,te p i q ortogonalne projekcije kateta a i b na hipotenuzu.

DOKAZ:



$$\vec{a}=\vec{v}-\vec{p}, \vec{b}=\vec{v}-\vec{q}$$

$$\vec{a}\cdot\vec{b}=0 \rightarrow (\vec{v}-\vec{p})\cdot(\vec{v}-\vec{q})=0 \rightarrow \vec{v}^2-\vec{v}\vec{q}-\vec{v}\vec{p}+\vec{p}\vec{q}=0$$

$$\vec{v}^2=|\vec{v}|^2=v^2, \vec{v}\vec{q}=0, \vec{v}\vec{p}=0, \vec{p}\vec{q}=|\vec{p}|\cdot|\vec{q}|\cdot\cos 180^\circ=-pq$$

$$\rightarrow v^2=pq \rightarrow v=\sqrt{pq}$$

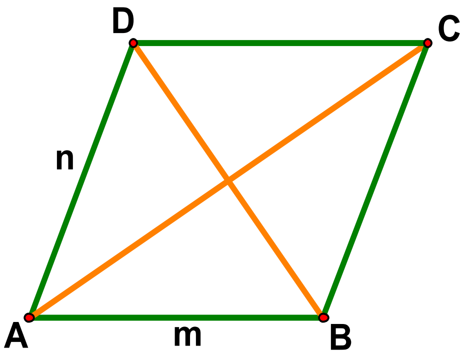
$$\vec{a}=\vec{v}-\vec{p} \rightarrow \vec{a}^2=\vec{v}^2-2\vec{v}\vec{p}+\vec{p}^2$$

$$\vec{a}^2=a^2, \vec{v}^2=v^2, \vec{v}\vec{p}=0, \vec{p}^2=p^2 \rightarrow$$

$$a^2=v^2+p^2=pq+p^2=p\cdot(q+p)=pc$$

$$\rightarrow a=\sqrt{pc},\ \vec{b}=\vec{v}-\vec{q} \rightarrow b=\sqrt{qc}$$

Poučak o dijagonalama romba:
Dijagonale romba međusobno su okomite.



DOKAZ:

$$\overrightarrow{AB}=\vec{m}, \overrightarrow{AD}=\vec{n}$$

$$|\vec{m}|=|\vec{n}|=a$$

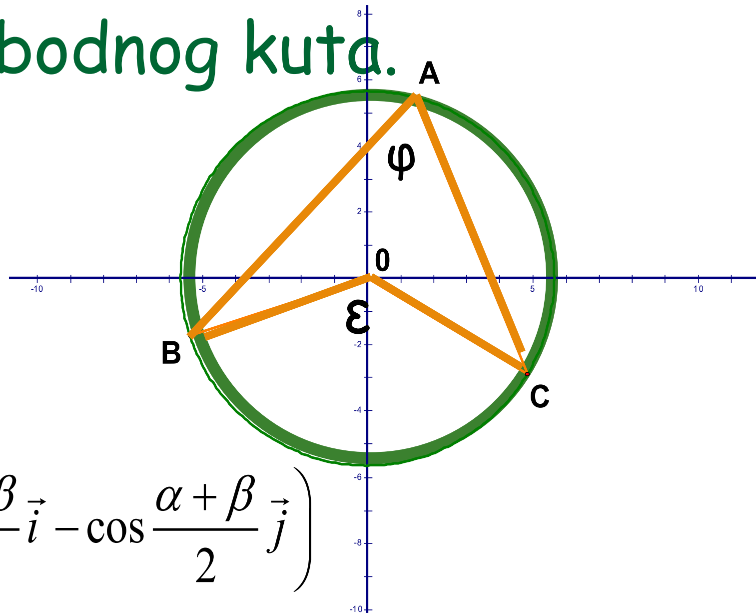
$$\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AB}+\overrightarrow{AD}=\vec{m}+\vec{n}$$

$$\overrightarrow{DB}=\overrightarrow{DA}+\overrightarrow{AB}=\vec{m}-\vec{n}$$

$$\overrightarrow{AC}\cdot\overrightarrow{BD}=(\vec{m}+\vec{n})\cdot(\vec{n}-\vec{m})=\vec{n}^2-\vec{m}^2=|\vec{n}|^2-|\vec{m}|^2=a^2-a^2=0$$

$$\rightarrow \overrightarrow{AC}\perp\overrightarrow{BD}$$

Poučak o obodnom i središnjem kutu: Središnji kut dvostruko je veći od pripadnog obodnog kuta.



DOKAZ:

$$A(\cos\alpha,\sin\alpha), B(\cos\beta,\sin\beta), C(\cos(\beta+\varepsilon),\sin(\beta+\varepsilon))$$

$$\overrightarrow{AB}=(\cos\beta-\cos\alpha)\vec{i}+(\sin\beta-\sin\alpha)\vec{j}=2\sin\frac{\alpha-\beta}{2}\left(\sin\frac{\alpha+\beta}{2}\vec{i}-\cos\frac{\alpha+\beta}{2}\vec{j}\right)$$

$$\overrightarrow{AC}=(\cos(\beta+\varepsilon)-\cos\alpha)\vec{i}+(\sin(\beta+\varepsilon)-\sin\alpha)\vec{j}=2\sin\frac{\alpha-\beta-\varepsilon}{2}\left(\sin\frac{\alpha+\beta+\varepsilon}{2}\vec{i}-\cos\frac{\alpha+\beta+\varepsilon}{2}\vec{j}\right)$$

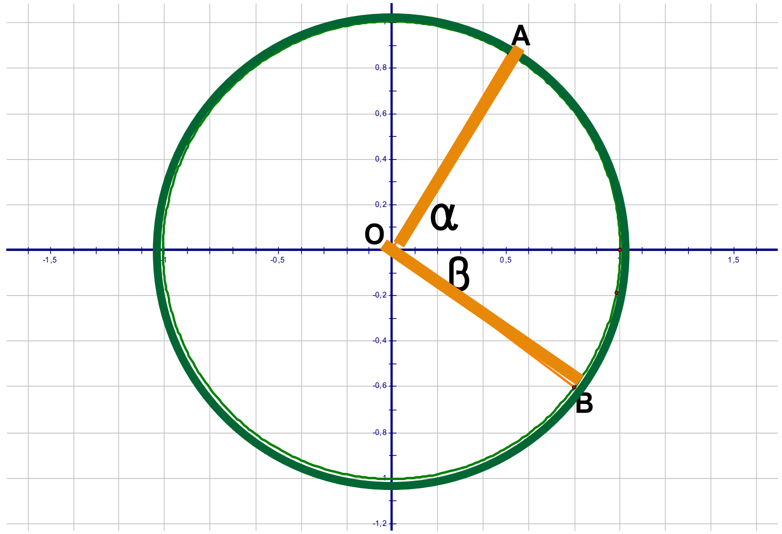
$$\cos\varphi=\frac{\overrightarrow{AB}\cdot\overrightarrow{AC}}{\left|\overrightarrow{AB}\right|\cdot\left|\overrightarrow{AC}\right|}=\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha+\beta+\varepsilon}{2}+\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha+\beta+\varepsilon}{2}=\cos\left(\frac{\alpha+\beta+\varepsilon}{2}-\frac{\alpha+\beta}{2}\right)=\cos\frac{\varepsilon}{2}$$

$$\rightarrow\varphi=\frac{\varepsilon}{2}$$

Adicijska formula za kosinus:

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$



DOKAZ:

$$\angle\left(\overrightarrow{OA},\overrightarrow{OB}\right)=\angle AOB=\alpha+\beta$$

$$\left|\overrightarrow{OA}\right|=\left|\overrightarrow{OB}\right|=1$$

$$\overrightarrow{OA}\cdot\overrightarrow{OB}=\left|\overrightarrow{OA}\right|\cdot\left|\overrightarrow{OB}\right|\cdot\cos\angle\left(\overrightarrow{OA},\overrightarrow{OB}\right)=\cos\left(\alpha+\beta\right)$$

$$A\left(\cos\alpha,\sin\alpha\right),B\left(\cos\beta,-\sin\beta\right)$$

$$\overrightarrow{OA}\cdot\overrightarrow{OB}=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$\rightarrow\cos\left(\alpha+\beta\right)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

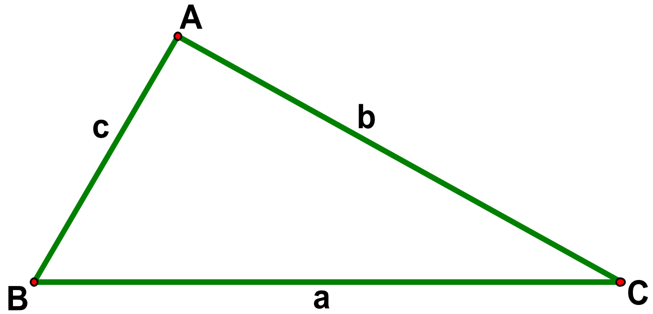
Poučak o kosinusu:

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

DOKAZ:



$$\vec{a} = \overrightarrow{BC}, \vec{b} = \overrightarrow{CA}, \vec{c} = \overrightarrow{BA}$$

$$\vec{a} = \vec{c} - \vec{b} \rightarrow a^2 = b^2 + c^2 - 2\vec{c}\vec{b}$$

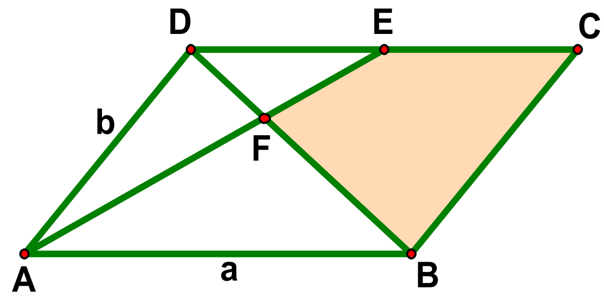
$$\vec{c}\vec{b} = c \cdot b \cdot \cos\alpha$$

$$\rightarrow a^2 = b^2 + c^2 - 2bc\cos\alpha$$

3

Zadatci

Zadatak 1: Pravac prolazi vrhom A i polovištem E stranice CD paralelograma ABCD, te siječe dijagonalu BD u točki F. Izračunajte površinu četverokuta BCEF, ako je površina paralelograma ABCD jednaka 24.



RJEŠENJE:

$$P_{BCEF}=P_{BCD}-P_{EDF}$$

$$P_{BCD}=\frac{1}{2}P_{ABCD}$$

$$P_{EDF}=\frac{1}{2}\left|\overrightarrow{DE}\times\overrightarrow{DF}\right|=\frac{1}{2}\left|\frac{1}{2}\vec{a}\times\lambda\overrightarrow{DB}\right|=\frac{\lambda}{4}\left|\vec{a}\times(\vec{a}-\vec{b})\right|=\frac{\lambda}{4}\left|\vec{a}\times\vec{b}\right|=\frac{\lambda}{4}P_{ABCD}$$

$$P_{EDF}=6\lambda$$

$$\overrightarrow{FE}=\mu\overrightarrow{AE}=\mu(\frac{1}{2}\vec{a}+\vec{b})=\frac{\mu}{2}\vec{a}+\mu\vec{b}$$

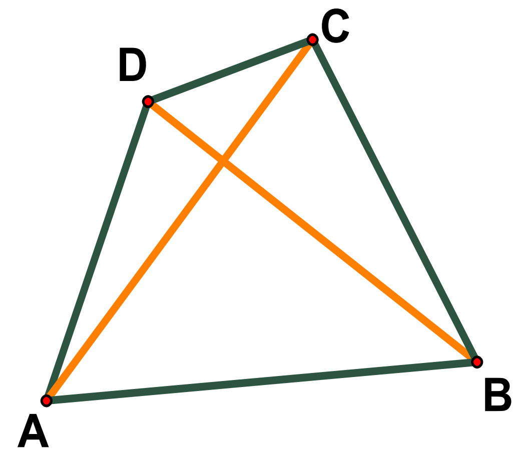
$$\overrightarrow{FE}=\overrightarrow{DE}-\overrightarrow{DF}=\frac{1}{2}\vec{a}-\lambda(\vec{a}-\vec{b})=(\frac{1}{2}-\lambda)\vec{a}+\lambda\vec{b}$$

$$\Rightarrow\quad \lambda=\frac{1}{3}\Rightarrow P_{EDF}=2\Rightarrow \underline{P_{BCEF}=10}$$

Zadatak 2: Dokaži da je površina četverokuta **ABCD** jednaka:

$$P=\frac{1}{2}\left|\overrightarrow{AC}\times\overrightarrow{BD}\right|$$

DOKAZ:



$$P=P_{ABC}+P_{ACD}=\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|+\frac{1}{2}\left|\overrightarrow{AC}\times\overrightarrow{AD}\right|$$

$$P=\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}+\overrightarrow{AC}\times\overrightarrow{AD}\right|$$

$$P=\frac{1}{2}\left|\overrightarrow{AC}\times\overrightarrow{BA}+\overrightarrow{AC}\times\overrightarrow{AD}\right|=\frac{1}{2}\left|\overrightarrow{AC}\times\left(\overrightarrow{BA}+\overrightarrow{AD}\right)\right|$$

$$\Rightarrow \qquad P=\frac{1}{2}\left|\overrightarrow{AC}\times\overrightarrow{BD}\right|$$

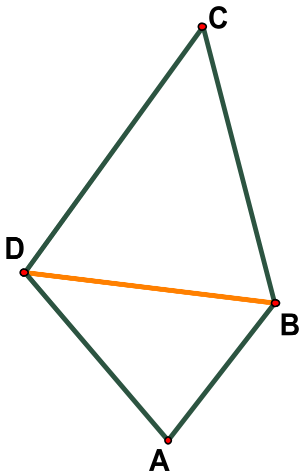
Zadatak 3: Bez uporabe kalkulatora odredi tangens kuta između dva vektora.

RJEŠENJE:

$$\varphi = \angle\left(\vec{a}, \vec{b}\right)$$

$$tg\varphi = \frac{\sin\varphi}{\cos\varphi} = \frac{\dfrac{\left|\vec{a} \times \vec{b}\right|}{\left|\vec{a}\right| \cdot \left|\vec{b}\right|}}{\dfrac{\vec{a} \cdot \vec{b}}{\left|\vec{a}\right| \cdot \left|\vec{b}\right|}} = \frac{\left|\vec{a} \times \vec{b}\right|}{\vec{a} \cdot \vec{b}}$$

Zadatak 4.: Zadani su vrhovi četverokuta ABCD, A(1,-7), B(3,-3), C(4,5), D(-2,-3).
Dokažite da dijagonala BD dijeli površinu četverokuta u omjeru 1:2.



RJEŠENJE:

$$\overrightarrow{DA}=3\vec{i}-4\vec{j},\overrightarrow{DB}=5\vec{i},\overrightarrow{DC}=6\vec{i}+8\vec{j}$$

$$P_{ABD}=\frac{1}{2}\cdot\left|\overrightarrow{DA}\times\overrightarrow{DB}\right|=\frac{1}{2}\cdot\left\|\begin{matrix}\vec{i} & \vec{j} & \vec{k}\\ 3 & -4 & 0\\ 5 & 0 & 0\end{matrix}\right\|=\frac{1}{2}\cdot 20\cdot\left|\vec{k}\right|=10$$

$$P_{BCD}=\frac{1}{2}\cdot\left|\overrightarrow{DB}\times\overrightarrow{DC}\right|=\frac{1}{2}\cdot\left\|\begin{matrix}\vec{i} & \vec{j} & \vec{k}\\ 5 & 0 & 0\\ 6 & 8 & 0\end{matrix}\right\|=\frac{1}{2}\cdot 40\cdot\left|\vec{k}\right|=20$$

$$\rightarrow P_{ABD}:P_{BCD}=1:2$$

Republičko natjecanje Bosne i Hercegovine 1. r. 1980.

Točka K središte je stranice $\overline{AB}$ kvadrata ABCD, a točka L dijeli dijagonalu $\overline{AC}$ u omjeru 3 : 1. Dokaži da je kut $\angle KLD$ pravi kut.

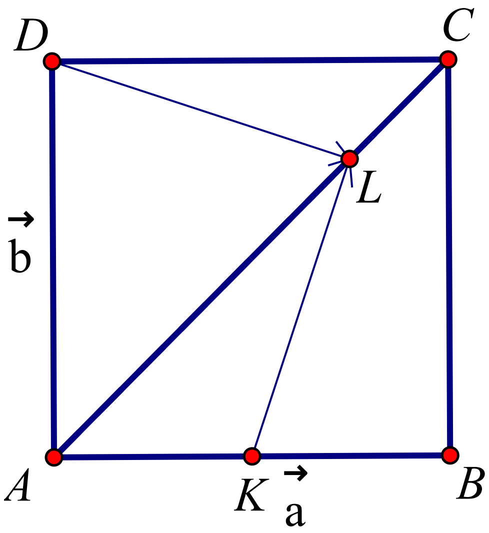
Neka je $\overrightarrow{AB} = \vec{a}$, $\overrightarrow{AD} = \vec{b}$.

$$\overrightarrow{KL} = -\frac{1}{2}\vec{a} + \frac{3}{4}(\vec{a} + \vec{b}) = \frac{1}{4}\vec{a} + \frac{3}{4}\vec{b}$$

$$\overrightarrow{DL} = -\vec{b} + \frac{3}{4}(\vec{a} + \vec{b}) = \frac{3}{4}\vec{a} - \frac{1}{4}\vec{b}$$

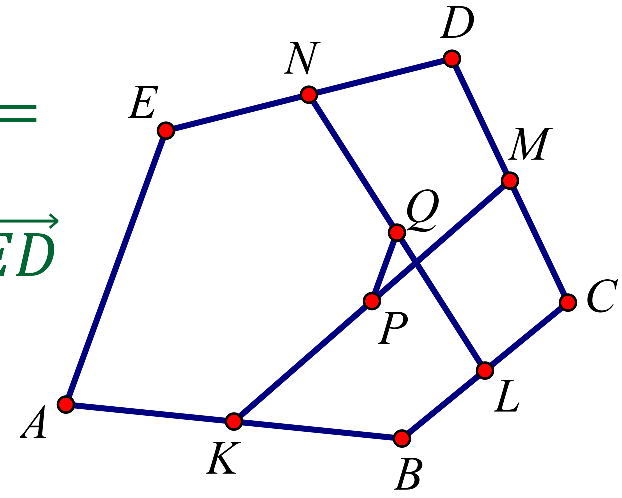
$$\overrightarrow{KL} \cdot \overrightarrow{DL} = \frac{1}{16}(\vec{a} + 3\vec{b})(3\vec{a} - \vec{b}) =$$

$$\frac{1}{16}\left(3|\vec{a}|^2 + 8\vec{a} \cdot \vec{b} - 3\left|\vec{b}\right|^2\right) = 0$$



Općinsko natjecanje Hrvatske 2. r 1989.

U konveksnom peterokutu ABCD točke K, L, M, N redom su polovišta stranica $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, a točke P i Q polovišta su dužina $\overline{KM}$ i $\overline{LN}$. Dokaži da je dužina $\overline{PQ}$ paralelna sa stranicom $\overline{AE}$ i da joj je duljina jednaka četvrtini duljine dužine $\overline{AE}$.



$$\overrightarrow{PQ} = \frac{1}{2}\overrightarrow{MK} + \frac{1}{2}\overrightarrow{BA} + \overrightarrow{AE} + \frac{1}{2}\overrightarrow{ED} + \frac{1}{2}\overrightarrow{NL} =$$

$$\frac{1}{2}\left(\frac{1}{2}\overrightarrow{DC} + \overrightarrow{CB} + \frac{1}{2}\overrightarrow{BA}\right) + \frac{1}{2}\overrightarrow{BA} + \overrightarrow{AE} + \frac{1}{2}\overrightarrow{ED}$$

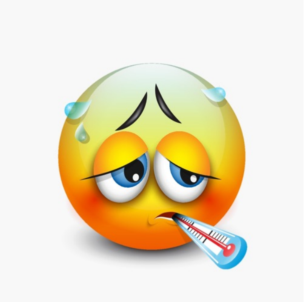
$$+ \frac{1}{2}\left(\frac{1}{2}\overrightarrow{ED} + \overrightarrow{DC} + \frac{1}{2}\overrightarrow{CB}\right)$$

$$= \frac{1}{4}\left(\overrightarrow{ED} + \overrightarrow{DC} + \overrightarrow{CB} + \overrightarrow{BA}\right) + \frac{1}{2}\left(\overrightarrow{ED} + \overrightarrow{DC} + \overrightarrow{CB} + \overrightarrow{BA}\right) + \overrightarrow{AE}$$

$$= \overrightarrow{AE} - \frac{3}{4}\overrightarrow{AE} = \frac{1}{4}\overrightarrow{AE}$$

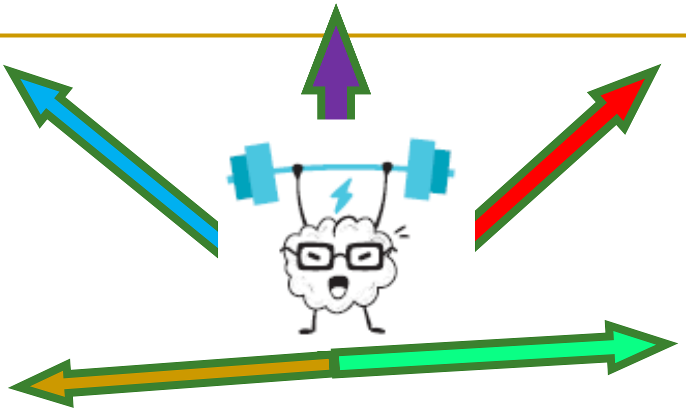
Umjesto zaključka

Vektorske zarazne bolesti



- Pojavljuju se od proljeća do jeseni
- Uzročnik – bakterija, virus, parazit
- Vektor – komarac, krpelj, stjenice
- Vanjski period inkubacije - vrijeme potrebno da vektor postane zarazan
- Domaćin – čovjek, životinja
- Zaraženi vektori najčešće doživotno prenose uzročnika bolesti
- Komarci –groznice (denga, žuta, čikungunja), malarija
- Krpelji – encefalitis, tifus, lajmska bolest
- Cijepljenje, komarnici, insekticidi

Vektorski trening

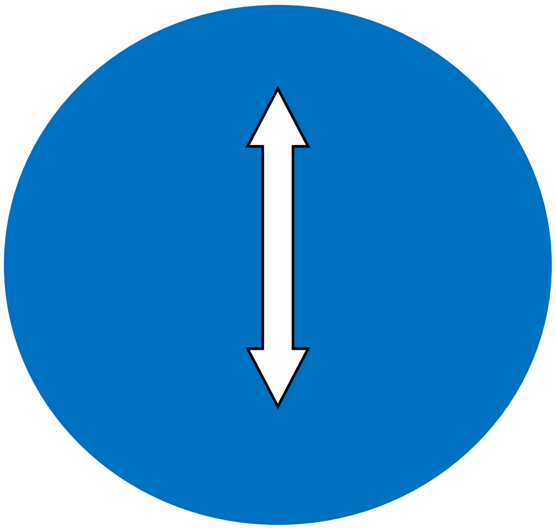


- Vektor – 5 traka različitih boja zalijepljenih na pod pokazuju smjer kretanja
- Kut 45 stupnjeva
- Duljina trake – duljina trupa
- Trake se dotiču rukama ili nogama
- Vektori poboljšavaju ravnotežu, stabiliziraju kukove i zdjelicu

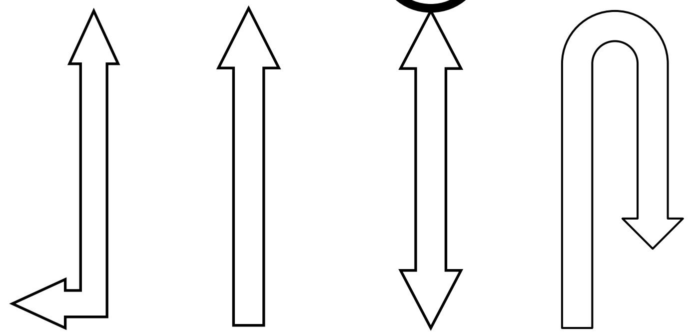
Istim smjerom

Matematičar polaže vozački ispit

Nadopuni znak za jednosmjernu ulicu

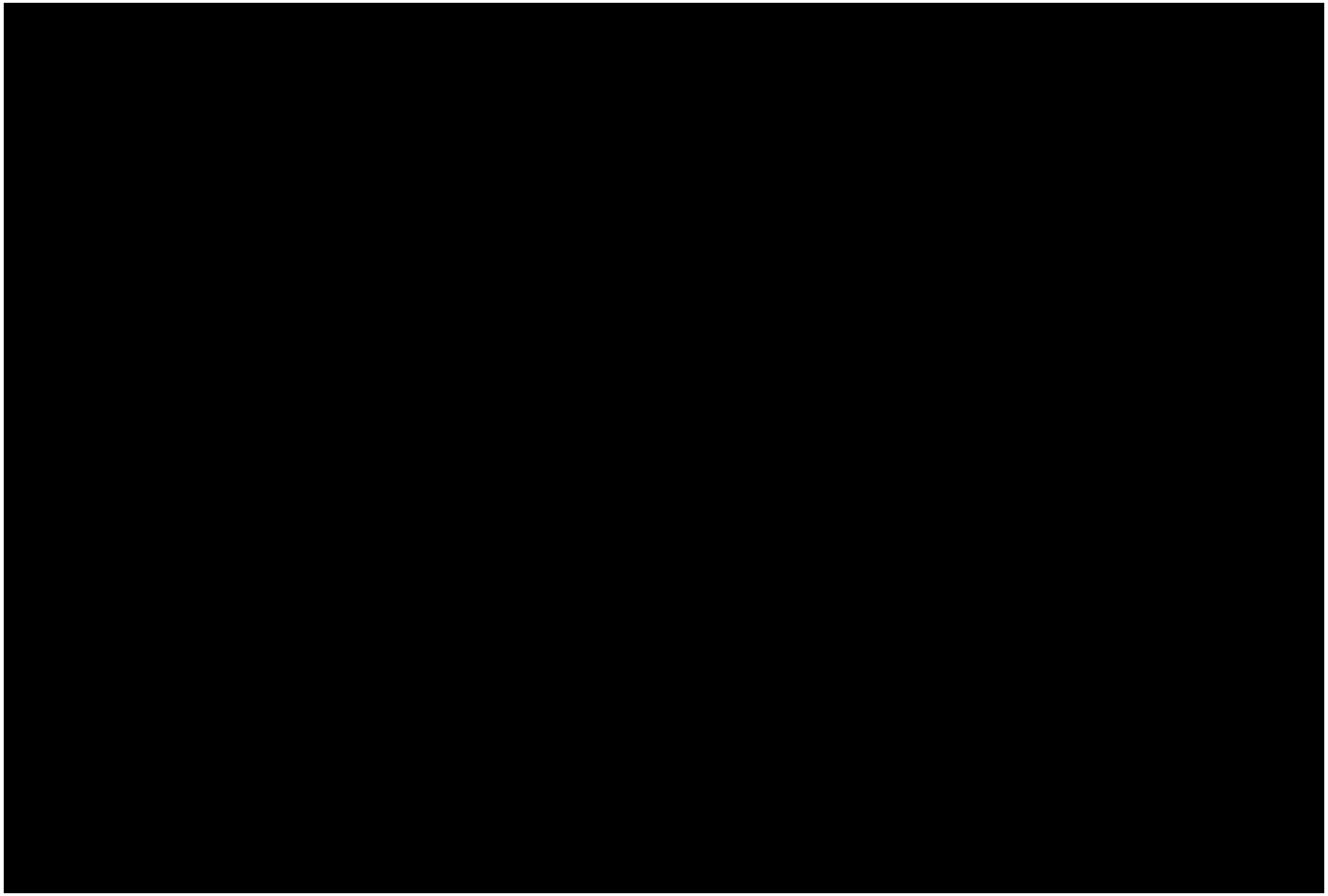


A. B. C. D.



Kako je Gru ukrao mjesec

https://www.youtube.com/watch?v=A05n32Bl0aY



Ja sam inače Vektor. To je matematički pojam, dužina predstavljena strelicom koja ima smjer i veličinu.

Vektor to sam ja, jer svi moji zločini imaju smjer i veličinu.

Hvala na pažnji!

KAVA
GORE
ISPRAVLJANJE
AMO
DOLJE
TAMO
RUČAK
ŠETNJA

Rebeka i Snježana